Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2021 – 2022

Calculators are allowed.

**Time: 10 minutes** **Name: _____**

7. Three boxes are on a scale. The heaviest of the three boxes weighs twice as much as the lightest of the three boxes. The lightest box weighs 2 pounds less than the second lightest box. The average weight of the three boxes is 30 pounds. Compute the weight of the heaviest box.

8. Kevin traveled from home to school. Rather than travel by bus, Kevin got a ride in a car for half the trip's distance. The car traveled at a steady rate, 12 times as fast as Kevin's walking rate. For the second half of the trip, Kevin had to take a bus and traveled at a steady rate only 3 times Kevin's walking rate. He completed the trip $c$ times as fast as he could if he had walked the entire trip. If, in simplest form, $c = \frac{p}{q}$, compute $p + q$.

7. ☐ 8. ☐

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9. A bag contains only red and green marbles. At first, there are 13 marbles in the bag. After 7 additional green marbles are inserted in the bag, the probability of randomly drawing a green marble from the bag is 55%. Compute the number of green marbles in the bag at first.

10. The equation $\frac{a-bi}{5}+\frac{-1+3i}{a+bi}=1$ has two complex number solutions, each of the form $a+bi$, where $i=\sqrt{-1}$. If those solutions are added and the resulting sum is written in the form $A+Bi$, compute $AB$.

9. ☐ 10. ☐

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**Time: 10 minutes** **Name: _____**

11. Compute $x$ if $7^x + 7^{x+1} = 134{,}456$.

12. Two circles are externally tangent. Their radii are 12 and 27. Compute the length of their common external tangent.

11. ☐ 12. ☐

**Solutions for Contest #2**

7. In increasing order, let the weights of the three boxes be $x, x+2$ and $2x$ pounds. So, $\frac{x+x+2+2x}{3} = 30 \rightarrow 4x+2 = 90 \rightarrow x = 22 \rightarrow 2x = \mathbf{44.}$

8. Use the numbers 12 and 3 from the problem. Without loss of generality, suppose the trip is 36 miles long and that his walking rate is 3 mph, the car's rate is 36 mph, and the bus's rate is 9 mph. If he had walked, his time for the entire trip would be 12 hours. Actually, he spent $\frac{18}{36} = \frac{1}{2}$ hour in the car and he spent another $\frac{18}{9} = 2$ hours on the bus. His actual travel time was 2.5 hours. Then $2.5 = 12c \rightarrow c = \frac{5}{24}$. The required sum is **29**. Alternatively, let $2d$ be the distance from home to school and let $r$ be Kevin's rate of walking. Then $12r$ and $3r$ are the rates of the car and bus, respectively. The total time traveled is $\frac{d}{12r} + \frac{d}{3r} = \frac{5d}{12r}$. It follows that $c\left(\frac{2d}{r}\right) = \frac{5d}{12r}$. So, $c = \frac{5}{24}$ and $5 + 24 = \mathbf{29}$.

9. When the marble is drawn, there are 20 marbles in the bag and 11 green marbles. So, initially there were $11 - 7 = \mathbf{4}$ green marbles in the bag.

10. The given equation can be re-written as $\frac{a^2+b^2-5+15i}{5(a+bi)} = 1 \rightarrow a^2 + b^2 - 5 + 15i = 5a + 5bi$. Set the imaginary parts equal to get $b = 3$ and substitute that value into the latter equation and re-arrange it to get $a^2 - 5a + 4 = 0 \rightarrow a = 4$ or 1. The solutions to the original equation are $4 + 3i$ and $1+3$i. Their sum is $5 + 6i$ and the required product is **30**.

11. $7^x + 7^{x+1} = 7^x(1+7) = 134{,}456 \rightarrow 7^x = 16807 \rightarrow x = \mathbf{5}$.

12. In the figure (not drawn to scale), circles $O$ and $P$ are tangent to each other at point $T$. Point $Q$ is on circle $O$, point $R$ is on circle $P$, $\overline{QR}$ is a common external tangent, $\overline{OQ}$ and $\overline{PR}$ are radii, and $\overleftrightarrow{OP}$ contains point $T$. Drop a perpendicular from point $O$ to $\overline{PR}$ with an intersection point $S$. Then, $\overline{OS} \perp \overline{PSR}$ and quadrilateral $QRSO$ is a rectangle. Therefore, $RS = QO = 12$ and $SP = 27 - 12 = 15$. Then the Pythagorean Theorem in right $\Delta POS$ yields $(OS)^2 = 39^2 - 15^2 \rightarrow OS = \mathbf{36} = QR$. (Note: 15-36-39 is a multiple of the Pythagorean triple 5-12-13).

